Appendix 9. Sample Growing Problem Solvers (Department) Article

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GPS: How Long? Estimating Length and Time

Growing Problem Solvers provides four original, related, classroom-ready mathematical tasks, one for each grade band. Together, these tasks illustrate the trajectory of learners' growth as problem solvers across their years of school mathematics.

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The focus of this month's Growing Problem Solvers is on using measurement to support number sense. These tasks invite students to conceptualize and discuss measurements, which we can think of as measurement sense! The tasks shared here purposefully focus on both physical measurement estimation and time estimation, so that the connections might support students' reasoning. Rather than focusing on specific units in the task, unit choice is left open for the students to negotiate and discuss. If needed, the teacher might prompt students to identify the unit of measurement and consider different unit options. At each grade band, teachers can launch and end the task by prompting student discussion with the following questions:

Table 1 Associated Common Core State Standards

Standard	Description
1.MD.A.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2.MD.A.3	Estimate lengths using units of inches, feet, centimeters, and meters.
3.MD.A.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
4.MD.A.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
HSG.GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
HSG.MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Appendix 9 (continued)

- When might it be reasonable to estimate a measurement?
- When is it better to get a precise measurement?
- How would you decide if your estimate is close enough?
- As a class, how do you decide if an estimation is acceptable? (What is your margin for error?)
- In the context of this problem, when would you rather underestimate? Overestimate?

Teachers might also consider including a discussion of comparison or reference objects (a hand, the size of a whiteboard, etc.) to support individuals or groups of students.

- What could you use as a reference when estimating?
- What other reference or comparison might be useful?
- What additional information do you need to make your estimate?

Table 1 states the Common Core State Standards for Mathematics (NGA Center and CCSSO 2010) addressed by the four tasks. The progression of the standards begins with one-dimensional measurements of length for the very early grades. Then area and perimeter (as an expansion on length) are introduced and developed in upper elementary grades and middle school grades. A special focus on various shapes is emphasized in the middle grades. Although the introductory conceptualizations of volume begin in the fifth grade and are further developed in the middle grades, this sequence of tasks focuses on three-dimensional shapes in the high school grade band.

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PK–2

In the PK–2 task (link online), the teacher begins the task by gathering a collection of small objects whose predominant attribute is length from around the classroom. These could be pencils, paper, markers, crayons, paintbrushes, paper clips, and so forth. This task could be completed by student pairs, with each pair having three objects. Students may intuitively consider wider objects (such as a piece of paper or a marker) as being longer. The teacher might need to emphasize the longer dimension through physical gestures and direct comparison of objects. The teacher might lead the students through a discussion using the following prompts:

- Which object do you think is the longest?
- Which object is the shortest?
- What makes you think so?
- How could you find out for sure?

Once the pairs answer these subquestions, the teacher would display another object and position it so that students could not line it up with their own objects. The aim here is for students to make estimations. Students may benefit from using a reference object, such as the teacher's hand, to form their estimations. Sharing various strategies would allow students to reflect on others' strategies and perhaps use those strategies later. Questions to lead this portion of the exploration include these:

- Is the teacher's object longer or shorter than your longest object?
- What makes you think so?
- How could you find out for sure?

As mentioned above, each task has one physical and one time estimation. We ask these young students to estimate how long it would take to count all the objects. Many students might think it would take the same number of seconds as the number of objects. The teacher could explore the students' reasoning with question such as the following:

- How long do you think it would take us to count all the objects we have gathered here?
- What makes you think so?
- How could we find out for sure?

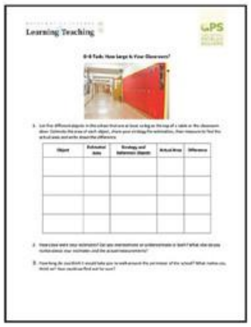
3–5

The upper elementary school task (link online) uses rectangular objects in the classroom, with a focus on perimeter. Students might confound perimeter and area, so the teacher may want to refer to the difference when launching the task. Some students might envision perimeter as one continuous length by picturing a rope going around the rectangular object and estimating the length all at once. Through shared strategies and questioning, the teacher might want to encourage the use of an estimation strategy that decomposes the rectangular object into tiles of smaller area, or by considering linear estimations of width and length first. For this task (and middle and high school tasks), students record their estimations, their estimation strategy, and their personal reference object (if they used one). Students then measure to find the actual perimeter and calculate the difference between their estimation and the actual measurement. This idea was inspired by the NCTM Illuminations Area Contractor lesson plan (Healy 2008). Keeping track of their estimations and the actual measurements could help students answer the second question and reflect on their estimation patterns. The time estimation for this task asks students to estimate how long it would take to walk around the perimeter of their classroom. The teacher and the class can also discuss how one could walk the "exact" perimeter of the classroom.

Appendix 9 (continued)

6–8

The task structure and questions for the middle school task (link online) are very similar to the upper-elementary task. For this grade band, the task focuses on area. Students will record their estimations and actual measurements to study their estimation patterns. With area estimation, students might estimate the length and width and then the area, or some students might use a reference object to tile the shape. Again, sharing and comparing different strategies might help students to consider other options for estimation. For the time estimation, middle schoolers are asked to estimate how long it would take to walk around the perimeter of the school. Even though this is not an estimation of time regarding area, it challenges middle school students to estimate the perimeter of the school.



9–12

The high school task (link online) focuses on volume and follows the same structure of the other tasks. High school students might estimate the length, width, and height to estimate the volume of an object, or they can think about filling a space with a reference object. This task asks students how long it would take to fill the volume of the largest object with water. This time estimation was inspired by Dan Meyer's (n.d.) Water Tank task. Watching Dan Meyer's video could help students adjust their time estimations.

REFERENCES

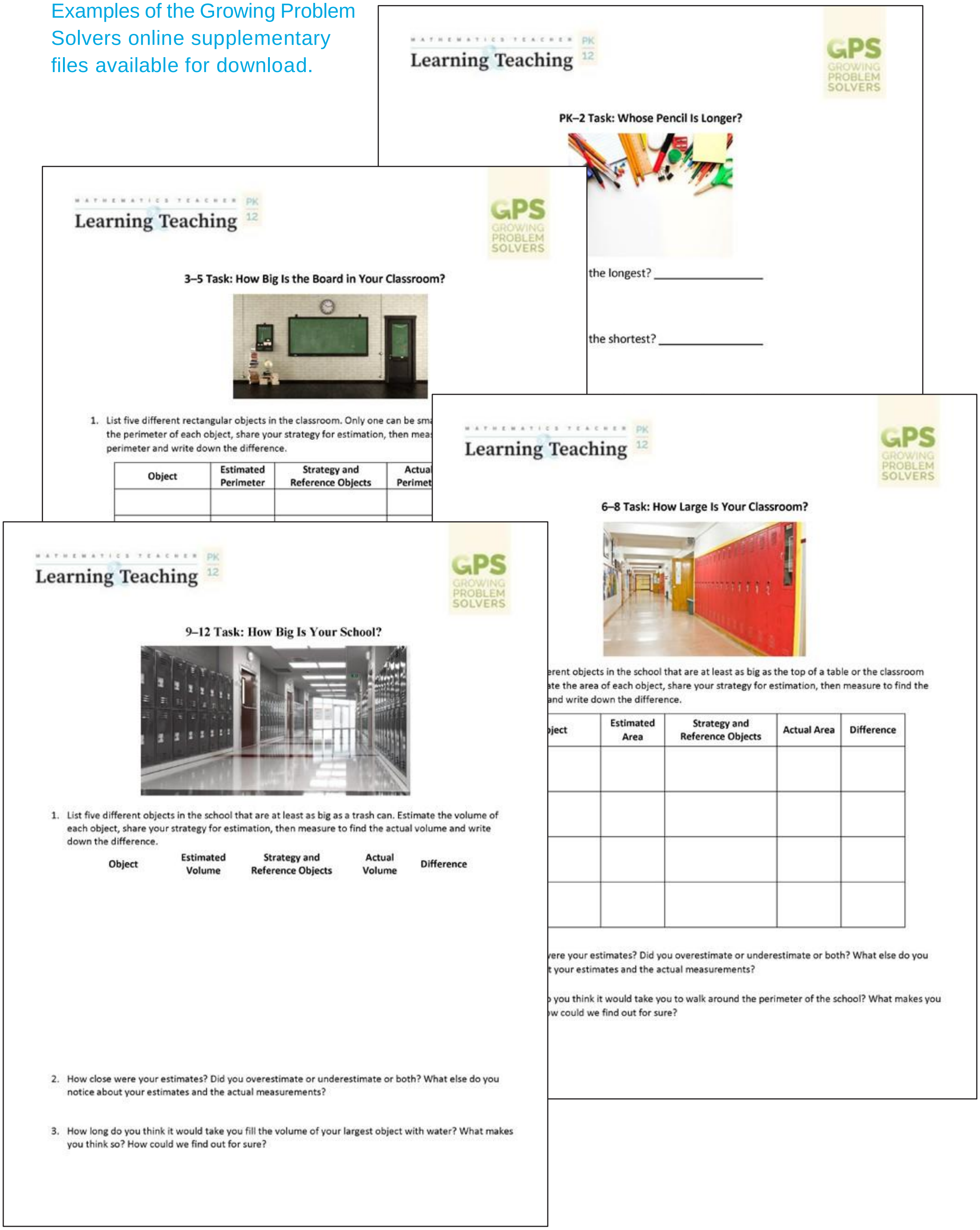
Healy, Julie. 2008. "Area Contractor." *Illuminations.* https://www.nctm.org/Classroom-Resources/Illuminations/Lessons/Area-Contractor/.

Meyer, Dan. n.d. "Water Tank." *Dan Meyer's Three-Act Math Tasks.* Accessed April 18, 2022. https://mrmeyer.com/threeacts/watertank/.

National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO). 2010. *Common Core State Standards for Mathematics.* Washington, DC: NGA Center and CCSSO. http://www.corestandards.org.

Appendix 9 (continued)

Examples of the Growing Problem Solvers online supplementary files available for download.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PK–2 Task: Whose Pencil Is Longer?

the longest? ______________

the shortest? ______________

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3–5 Task: How Big Is the Board in Your Classroom?

1. List five different rectangular objects in the classroom. Only one can be sma… the perimeter of each object, share your strategy for estimation, then mea… perimeter and write down the difference.

Object	Estimated Perimeter	Strategy and Reference Objects	Actual Perimet…

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6–8 Task: How Large Is Your Classroom?

…erent objects in the school that are at least as big as the top of a table or the classroom …ate the area of each object, share your strategy for estimation, then measure to find the …nd write down the difference.

…ject	Estimated Area	Strategy and Reference Objects	Actual Area	Difference

…ere your estimates? Did you overestimate or underestimate or both? What else do you …t your estimates and the actual measurements?

…you think it would take you to walk around the perimeter of the school? What makes you …w could we find out for sure?

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9–12 Task: How Big Is Your School?

1. List five different objects in the school that are at least as big as a trash can. Estimate the volume of each object, share your strategy for estimation, then measure to find the actual volume and write down the difference.

Object	Estimated Volume	Strategy and Reference Objects	Actual Volume	Difference

2. How close were your estimates? Did you overestimate or underestimate or both? What else do you notice about your estimates and the actual measurements?

3. How long do you think it would take you fill the volume of your largest object with water? What makes you think so? How could we find out for sure?