Appendix 3. Sample Feature Article

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Eight Unproductive Practices in Developing Fact Fluency

Basic fact fluency has always been of interest to elementary school teachers and is particularly relevant because a wide variety of supplementary materials of varying quality exist for this topic. This article unpacks eight common unproductive practices with basic facts instruction and assessment.

Gina Kling and Jennifer M. Bay-Williams

Ensuring students master their basic facts is a shared priority among teachers, parents, and administrators. As a result, the number of available resources for basic facts instruction, from concrete to virtual, is vast and continues to grow. Yet, many resources do not reflect the vision expressed in *Catalyzing Change in Early Childhood and Elementary Mathematics* (NCTM 2020) for ensuring each and every student has access to mathematics. In line with NCTM recommendations, basic fact fluency can and must be developed in mathematics environments that emphasize "curiosity, flexibility, and wonder" (NCTM 2020, p. 82). Long-standing methods for teaching basic facts have not been effective for far too many students. Now, more than ever, it is important not to succumb to the allure of "quick fix" programs like rote drill online applications. Instead, it is time to acknowledge common unproductive practices and look to effective alternatives.

UNPRODUCTIVE PRACTICE 1

Not encouraging every student—including those who already know their facts—to learn reasoning strategies

Example. A student uses a finger-trick to recall their 9s facts but does not learn a strategy such as subtracting a group from the related 10s fact (e.g., 9 × 7 = 10 × 7 − 7).

Why it is unproductive. First, tricks like the finger patterns may get to an answer, but they deny students an opportunity to reason quantitatively. Learning the Subtract-a-Group strategy not only helps students learn their 9s facts but also extends to different types of numbers. For example—

- larger values: 99 × 7 → think 100 groups of 7 is 700 and just subtract a group of 7 to get 693.
- decimal values: 7.9 × 5 → think 8 × 5 = 40 and just subtract 0.5 (one-tenth of 5) to get 39.5.

In both of these examples, the standard algorithm for multiplication is less efficient than reasoning.

Second, *procedural fluency* includes three components: (1) flexibility, (2) efficiency, and (3) accuracy (NCTM 2014b; NRC 2001). When basic facts are learned through tricks or rote practice and memorization, students do not develop flexibility or efficiency; hence, they do not develop basic fact *fluency*.

Instead. Focus instruction on strategy development. Students need opportunities to reason and access to a variety of efficient strategies. This sets them up for success with procedural fluency for all operations and number types. Table 1 shows how just two addition fact strategies extend to other numbers. Given how useful these two strategies are, what a significant mistake it is for students to not engage with these strategies as they learn their basic facts!

UNPRODUCTIVE PRACTICE 2

Telling a strategy rather than providing explicit strategy instruction

Example. A teacher introduces the Making 10 strategy by telling students to take some from the smaller addend to give to the larger addend before adding. For example, for 8 + 7, take 2 from the 7 to give to the 8 to make 10, and then add the leftover 5 to get 15.

Why it is unproductive. You cannot simply tell a student to understand. Many studies show that students who use strategies with understanding outperform their peers (Baroody et al. 2016; Locuniak and Jordan 2008; Purpura et al. 2016; Tournaki 2003). Students need time and experiences to see number relationships and develop understanding, exploring representations and connecting them to abstract reasoning (Clements, Fuson, and Sarama 2017; NRC 2009; Van de Walle, Karp, and Bay-Williams 2019).

Instead. Use visuals and contexts in ways that lead students to be the ones discovering and explaining the strategies (how they work and why they work). For each and every child to understand and be able to use strategies, instruction must be explicit. Merriam-Webster.com defines *explicit* as "fully revealed or expressed without vagueness." One way to reveal a strategy is to use Quick Looks that nudge children to develop a particular strategy (Bay-Williams and Kling 2019). A Quick Look activity involves showing an image briefly (two to three seconds) and then hiding it, asking children to use visualization strategies to describe what they saw. After providing a second look, the class discusses both the number of dots shown and most importantly, *how students saw it*. Carefully pairing Quick Looks can help students use known facts to employ a strategy. Figure 1 shows two examples of paired Quick Looks, one that works for both Making 10 and Pretend-a-10 (addition) and one for Doubling (multiplication). Please see video 1 (link online) for a demonstration on using Quick Looks.

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Table 1 Basic Fact Strategies Grow into General Reasoning Strategies

Basic Fact Reasoning Strategy	Extended to Multidigit Numbers	Extended to Rational Numbers
Making 10: **9 + 6 → 10 + 5** (Move a quantity from one addend to the other to make a 10.)	*Make Tens:* 69 + 58 = 70 + 57 = 127 *Make Hundreds:* 395 + 784 = 400 + 779 = 1,179	*Make a whole:* 4 7/8 + 3 5/8 = 5 + 3 4/8 = 8 1/2 8.9 + 6.2 = 9 + 6.1 = 15.1
Pretend-a-10/use 10: **9 + 6 → 10 + 6** **16 – 1 = 15** (Pretend the 9 is a 10, add, take one off the answer because 9 is one less than 10.)	*Compensation:* *Adjust both numbers:* 39 + 28 → 40 + 30 – 3 → 70 – 3 → 67 *Adjust one number:* 398 + 514 → 400 + 514 – 2 → 914 – 2 → 912	*Compensation:* *Adjust one number:* 5.9 + 6.47 → 6 + 6.47 – 0.1 → 12.47 – 0.1 → 12.37 *Adjust both numbers:* 7 7/8 + 12 7/8 → 8 + 13 – 2/8 → 20 3/4

Adapted from SanGiovanni and colleagues (2022).

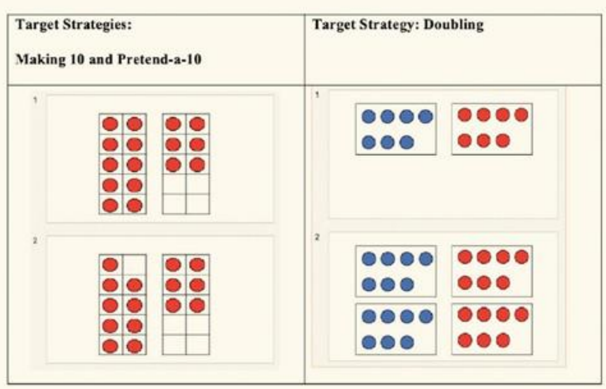
We have found that when we use Quick Looks, the thinking behind the strategy emerges from students as they describe how they determined the total. As teachers, we can attend to that strategy and invite students to try it on a new Quick Look. After several Quick Looks with a similar theme, the teacher can close the discussion by engaging the class in summarizing and classifying the strategy (i.e., "All of our cards had one ten-frame close to a 10, which made it easier to make 10. So, our Making 10 strategy works well when an addend is close to 10"). In this way, we make strategies explicit while still preserving the ownership of the strategy by the students.

UNPRODUCTIVE PRACTICE 3

Teaching facts in order of addend or factor size (0s, then 1s, then 2s, etc.)

Example. Having a mastery progress chart, with columns 0–10 is an example of this unproductive practice.

Fig. 1



This figure shows paired Quick Looks to reveal strategies.

Appendix 3 (continued)

This is how most adults remember learning multiplication facts!

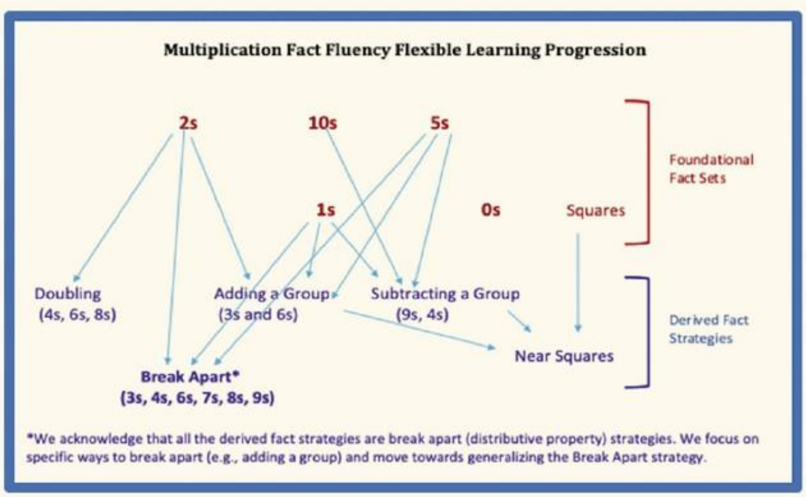
Why it is unproductive. The traditional order treats facts as isolated objects, does not build on students' strengths and prior knowledge, and can result in lower achievement (Brendefur et al. 2015; Henry and Brown 2008; Steinberg 1985; Thornton 1978).

Video 1 A Quick Looks Demonstration

Watch the full video online.

Instead. Use a research-based, strategy-focused progression (e.g., Baroody 2006; Brendefur et al. 2015; Heege 1985; Steinberg 1985; Thornton 1978). Figure 2 illustrates such a learning progression for multiplication. It begins with two fact groups most familiar to students and also vital to the strategies they will learn: the 2s and 10s. Next are the 5s. Although students can skip-count by 5s, learning instead to take half of the related 10s fact can better encourage fluency. Then with *groups* understood, 1 group (and 0 groups) make sense, and the 1s and 0s can be mastered with meaning. Next introduce the squares, which are not even represented in a traditional go-in-numerical-order approach, but are a very helpful fact group to know because some of the toughest facts to learn (e.g., 7 × 8 and 6 × 7) are close to squares, and squares are useful for later work in algebra, geometry, and measurement. Note the trajectory below is divided into "foundational facts" and "derived facts." The most important aspect of this progression is that foundational facts must be mastered before engaging students in making sense of the Derived Fact strategies. We invite the reader to examine figure 3 and consider how foundational facts are used in the given samples of student work from end-of-year third graders. Such examples illustrate the necessity of mastering foundational facts for developing reasoning strategies.

Fig. 2



This learning progression for multiplication facts demonstrates a research-based, strategy-focused progression. Adapted from Bay-Williams and Kling 2019.

UNPRODUCTIVE PRACTICE 4

Not using a coherent, multigrade approach to facts instruction

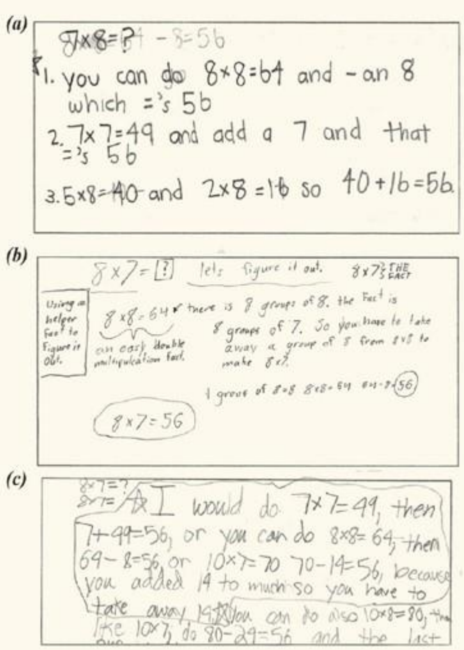
Examples. A first-grade teacher presses for automaticity within 10 at the expense of working on reasoning strategies within 20. A third-grade class focuses on rote memorization as a priority over learning strategies.

Why it is unproductive. This approach does not work! Too often, teachers at a particular grade level feel pressured to ensure their students have mastered their basic facts by the end of the school year. And yet year after year, *facts memorized through rote techniques do not tend to stick.* As a result, "many educators find that children, even in the upper grades, continue to draw tally marks and count by ones as their dominant solution approach in solving problems" (NCTM 2020, p. 82).

Instead. Be the tortoise in the fable of the tortoise and the hare. It is better for students' future learning and their mathematical identities to focus on learning strategies and not feel pressured to memorize. Ensuring students have developed automaticity with foundational facts before moving on to strategy instruction is an important aspect of pacing. Key to this is collaboration across grade levels to create a coherent facts mastery plan grounded in research-based learning progressions (see figure 2 for multiplication and figure 4 for addition). A K–2 plan might elaborate on when the elements of the addition learning progression are developed and when we may reasonably expect automaticity. For example, you may have the goal of "automaticity of all foundational facts by the end of grade 1." Such a goal helps set up children for success in mastering strategies throughout grade 2 for both addition and subtraction, with the goal of automaticity with addition facts by the end of grade 2.

Mastering multiplication facts often appears as a third-grade standard—a tall order to be sure! To help ease this challenge, children can actually begin exploring multiplication at the end of second grade, learning multiplication through story contexts that involve equal groups (e.g., *A box of granola bars contains five bars. If there are three boxes on the shelf, how many granola bars are there?*). Spending time in grade 2 solving story problems as well as on Quick Looks involving equal groups and arrays (see figure 1 and video 1 [link online]) can help children gain a healthy, developmentally appropriate jump on their learning of multiplication facts.

Fig. 3



These sample responses from third graders to the question, "If your friend didn't know the answer to 7 × 8, how could they figure it out?" illustrate the necessity of mastering foundational facts in developing reasoning strategies.

UNPRODUCTIVE PRACTICE 5

Teaching only the think addition strategy for subtraction facts

Example. Some teachers instruct students to change every subtraction fact to the related addition fact (e.g., change 15 – 9 = ? to 9 + ? = 15) and ask them to recall the addition fact.

Why it is unproductive. Other subtraction strategies warrant attention because they can transfer to computation beyond facts (Steinberg 1985).

Instead. Teach Think Addition *and* other strategies. There is no question that the inverse relationship between addition and subtraction is a powerful tool to use when developing computational fluency. Research suggests that children tend to be more accurate with addition than subtraction (NRC 2001). Thus, thinking of the related addition facts does build on students' strengths. But balancing this with other strategies supports reasoning and prepares students for work beyond facts. Let's revisit the example of 15 – 9 and look at other options:

1. Use compensation: Start with 15 – 10 = 5 then compensate by adding 1 back to get 6.
2. Break apart the subtrahend to go down under 10: 15 – 5 = 10 and then 10 – 4 = 6.
3. Break apart the minuend and subtract from 10: 10 – 9 = 1 and then 5 + 1 = 6.

Such strategies prepare students to use similar methods with other numbers. For example, for 132 – 99, a student may think 132 – 100 and compensate by adding 1, or they may break apart the minuend to 100 + 32, subtract 99 from 100 (1) and add that to 32 to get 33. Just as with addition facts, providing students with opportunities to learn a variety of subtraction strategies has value far beyond fact work.

UNPRODUCTIVE PRACTICE 6

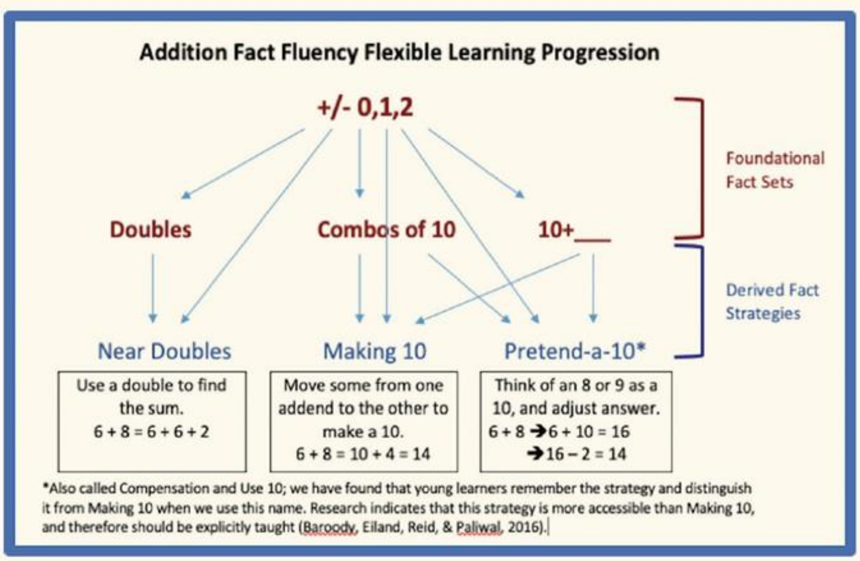
Using traditional forms of fact practice

Example. Using activity sheets that focus on a fact group (e.g., multiplying by 6).

Why it is unproductive. Activity sheets assess only accuracy and thus do not focus on fluency. The message such a practice sends to students is that the memorization of isolated facts is what is important, rather than developing, discussing, and applying strategies. Furthermore, such a practice is distasteful to students. Very few students want to do 30+ problems on a page. That bad taste leads many students to decide they do not like mathematics.

Instead. Fact mastery can be achieved by meaningful experiences with physical games and interactive activities.

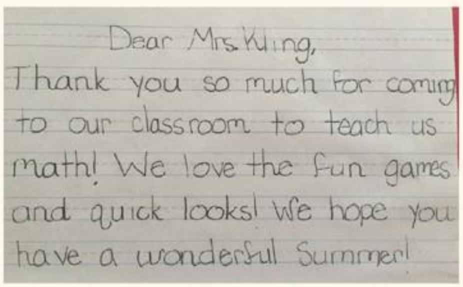
Fig. 4



This addition learning progression represents a facts mastery plan grounded in research-based learning progressions. Adapted from Bay-Williams and Kling 2019.

Unlike activity sheets, computer applications, or flash cards, games offer valuable opportunities for discussion and reflection. Games can focus on a single fact set (e.g., combinations of 10) or a strategy (e.g., doubling), providing developmental support for students. Teachers can use observation during game play to gather more authentic assessment data that address all components of fluency: efficiency, flexibility, accuracy, and appropriate strategy use. Playing facts games with classmates or family members is one of the most beneficial ways to encourage children to master their basic facts and can greatly increase their taste for mathematics, as shown in figure 5.

Fig. 5


Dear Mrs. Kling,
Thank you so much for coming to our classroom to teach us math! We love the fun games and quick looks! We hope you have a wonderful Summer!

An end-of-year thank you note from second graders expresses appreciation for the author's use of facts games.

UNPRODUCTIVE PRACTICE 7

Pressing for speed

Example. Practicing facts by doing a board race or other game in which speed is a factor, including many online applications.

Why it is unproductive. Requiring children to generate answers quickly before they have sufficiently developed and practiced strategies has a tendency to drive fluency development in the opposite direction (i.e., back to counting or skip counting) because mentally implementing a strategy initially takes more time and concentration. In summary, "Fluency does not equal speed" (NCTM 2020, p. 89).

Instead. Choose games that have each student solving a different problem as they take turns. For example, first graders can play a modified version of the classic game Go Fish, in which players look for combinations of two cards that sum to 10 instead of matching pairs. This widely-used game is highly enjoyable for students and provides low-stress, meaningful practice of an important facts group. For multiplication, TRIOS (see figure 6) is one of our favorite games and is easily adapted to other multiplication fact sets or even addition combinations (Bay-Williams and Kling 2019). Note that when you use the TRIOS game board, you are also

Fig. 6

TRIOS: Multiples of Six

18	54	30	0	42
36	24	12	18	48
6	42	0	48	30
48	18	36	12	24
24	6	54	42	36

Materials: 10-sided die [you can also use cards], game board, two different colored counters (or two colors of marker).

Goal: Get as many 3-in-a-row as possible (they can overlap).

How to Play: Player 1 rolls the die, multiplies it by 6 and chooses a square to claim on the game board. Player 2 takes a turn. Players can go for their own 'trios' or block their opponent. Play until gameboard is full (or time is up). Players count up their 'trios', multiply that number by 6, and high score wins.

This is an example of the TRIOS game with no speed element.

infusing ideas of division as students think of the factor they hope to draw or roll.

UNPRODUCTIVE PRACTICE 8

Using timed testing to assess fluency

Example. Asking third graders to solve 100 multiplication problems in three minutes can be counterproductive.

Why it is unproductive. Timed testing has a long history in American mathematics education, and it is likely that one does not need to look far to find a classroom that still uses it. Despite the history, many reasons exist for why this practice must be eradicated. First of all, if schools value true fluency as defined earlier in this article, one must immediately recognize that timed testing cannot possibly assess flexibility or strategy use. That is not to say children are *not* flexible, or that they are *not* using an appropriate strategy; a timed test simply does not assess these things.

Second, the two parts of fluency that timed tests are supposed to measure (efficiency and accuracy) in truth are not reliably measured. There is nothing stopping students from quickly counting, even though it is not an efficient method. Furthermore, fine motor capabilities in young children affect how quickly they can record answers, thus interfering with efficiency and completion. Finally, the anxiety that many students experience when taking a timed test can hamper their abilities to think clearly, causing them to underperform in such settings (Boaler, Williams, and Confer 2015). A growing body of research suggests that mathematics anxiety starts as early as first grade and can have permanent impacts, including mathematics avoidance in adults (Choe et al. 2019; Ramirez et al. 2013).

Instead. Many assessment options do not have the negative impact of timed testing and actually offer better assessment data. They include observations, interviews, writing samples, strategy sorts, and self-assessments (Bay-Williams and Kling 2019; Kling and Bay-Williams 2014). Interviewing is a particularly powerful (and underused) tool, despite an abundance of advocacy to focus on children's mathematical thinking (Jacobs, Lamb, and Philipp 2010; NCTM 2014a; NRC 2009). At its most basic level, interviewing involves asking a student two simple questions:

> "What is [8 × 7]?"
> "How did you figure it out?"

The follow-up question allows the interviewer to note which strategies the student is using and to communicate to the student *that their thinking matters.* A short list of problems (10 or fewer) can give you enough data to assess strategy use, flexibility, accuracy, and efficiency. Automaticity, which is often defined as within three seconds, can also be assessed as interviewers silently count in their heads to determine whether the student is automatic without them knowing they are being timed. Teachers who have experienced interviews see that they are more effective and affirming, as the following quotes (collected by the first author) illustrate:

- Because if you just gave them a timed test, like you wouldn't know, um, where their struggle was at least when you're working with them you can kind of see, can they apply a strategy or if they're trying to apply a strategy, where is it going wrong?
- So it was really exciting to see what was happening and then know, here's that entry point right now to help that student.
- I just like, I absolutely love it. I just, I just love seeing the, the strategies that kids use and love, it's just so cool to see and I never would have gotten to see that had I not taken the time to pull the kids, you know, in the hallway to talk to them about it.

CONCLUDING REMARKS

We (the authors) are not "glass-half-empty" people. Yet, we have seen how these unproductive moves have had tragic consequences, denying students access to research-based, effective teaching practices; inhibiting their attainment of reasoning strategies; and shaping their mathematical identities in negative ways. As a result, society loses potential mathematicians, scientists, engineers, and more. We hope that our "instead" ideas present steps in the right direction. Inviting parents into the conversation is also important—share why you are avoiding the traditional practices above and discuss the value of learning strategies with respect to long-term mathematical reasoning and their child's emotional connection to mathematics. Use learning progressions and diagnostic assessment practices that will lead to efficient teaching of the basic facts. As you make thoughtful decisions about your next steps in supporting your students, consider the vital importance of basic fact fluency, with an eye on the long-term goal of creating thoughtful, creative, and flexible mathematical thinkers.

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